



Control Authority of a Mortar Using Internal Translating Mass Control

by Jonathan Rogers and Ilmars Celmins

ARL-TR-4857

June 2009

NOTICES

Disclaimers

The findings in this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

Citation of manufacturer's or trade names does not constitute an official endorsement or approval of the use thereof.

Destroy this report when it is no longer needed. Do not return it to the originator.

Army Research Laboratory

Aberdeen Proving Ground, MD 21005-5066

ARL-TR-4857**June 2009**

Control Authority of a Mortar Using Internal Translating Mass Control

Jonathan Rogers
Georgia Institute of Technology

Ilmars Celmins
Weapons and Materials Research Directorate, ARL

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing the burden, to Department of Defense, Washington Headquarters Services, Directorate for Information Operations and Reports (0704-0188), 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302. Respondents should be aware that notwithstanding any other provision of law, no person shall be subject to any penalty for failing to comply with a collection of information if it does not display a currently valid OMB control number. PLEASE DO NOT RETURN YOUR FORM TO THE ABOVE ADDRESS.					
1. REPORT DATE (DD-MM-YYYY) June 2009		2. REPORT TYPE Final		3. DATES COVERED (From - To) December 2008–February 2009	
4. TITLE AND SUBTITLE Control Authority of a Mortar Using Internal Translating Mass Control				5a. CONTRACT NUMBER	
				5b. GRANT NUMBER	
				5c. PROGRAM ELEMENT NUMBER	
6. AUTHOR(S) Jonathan Rogers * and Ilmars Celmins				5d. PROJECT NUMBER 1L162618AH80	
				5e. TASK NUMBER	
				5f. WORK UNIT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) U.S. Army Research Laboratory ATTN: AMSRD-ARL-WM-BC Aberdeen Proving Ground, MD 21005-5066				8. PERFORMING ORGANIZATION REPORT NUMBER ARL-TR-4857	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)				10. SPONSOR/MONITOR'S ACRONYM(S)	
				11. SPONSOR/MONITOR'S REPORT NUMBER(S)	
12. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution is unlimited.					
13. SUPPLEMENTARY NOTES *Georgia Institute of Technology, Department of Aerospace Engineering, 270 Ferst Dr., Atlanta, GA 30332					
14. ABSTRACT This report examines the control authority of an 81-mm mortar projectile equipped with an internal translating mass. The translating mass undergoes controlled motion within the projectile body, resulting in a lateral mass center offset that creates a drag-induced moment. This moment can be utilized to guide the round to a target. Several example studies demonstrate basic control authority characteristics, while trade studies examine the effect of projectile static stability as well as mass size, oscillation amplitude, and location on overall maneuver capability.					
15. SUBJECT TERMS projectile, guidance, mortar, translating mass					
16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT UU	18. NUMBER OF PAGES 28	19a. NAME OF RESPONSIBLE PERSON Ilmars Celmins
a. REPORT Unclassified	b. ABSTRACT Unclassified	c. THIS PAGE Unclassified			19b. TELEPHONE NUMBER (Include area code) (410) 306-0781

Contents

List of Figures	iv
List of Tables	v
1. Introduction	1
2. Dynamic Model	1
2.1 Dynamic Equations	3
2.2 Example Projectile	5
2.3 Internal Translating Mass Control Setup.....	6
3. Results	7
3.1 Example Trajectories.....	7
3.2 Trade Studies	11
4. Conclusion	13
List of Symbols, Abbreviations, and Acronyms	15
Distribution List	17

List of Figures

Figure 1. Top down view of 81-mm mortar equipped with an ITM.....	2
Figure 2. Position coordinates schematic of the 81-mm mortar projectile.	2
Figure 3. Moving mass position, velocity (\vec{v}) and acceleration (\vec{a}) over one roll cycle, viewed from the rear of the projectile. Mass motion is sinusoidal and has the same frequency as the projectile roll rate.....	6
Figure 4. Altitude vs. range. All trajectories overlay one another.	8
Figure 5. Deflection vs. range.....	8
Figure 6. Total velocity vs. time. All trajectories overlay one another.	9
Figure 7. Roll rate vs. time. All trajectories approximately overlay one another.	9
Figure 8. Zoom view of roll rate vs. time. Notice roll rate oscillations as ITM oscillates, changing the axial moment of inertia of the system.	10
Figure 9. Segment of ITM displacement from centerline vs. time.	10
Figure 10. Angle of attack vs. time.....	11
Figure 11. Radial deviation from standard trajectory vs. cavity offset for nominal stability (ITM mass of 125 g, CG stationline of 0.282 m).....	12
Figure 12. Radial deviation from standard trajectory vs. cavity offset for reduced stability (ITM mass of 125 g, CG stationline of 0.209 m).....	12
Figure 13. Radial deviation from standard trajectory vs. ITM stroke for nominal stability (cavity offset of 0.167 m, CG stationline of 0.282 m).....	13
Figure 14. Radial deviation from standard trajectory vs. ITM stroke for reduced stability (cavity offset of 0.167 m, CG stationline of 0.209 m).....	14

List of Tables

Table 1. Properties of the example mortar projectile.....	5
Table 2. Initial conditions for the example mortar projectile.	5
Table 3. ITM configuration parameters for example trajectories.	6

INTENTIONALLY LEFT BLANK.

1. Introduction

This report examines basic control authority characteristics of an example 81-mm mortar equipped with a controllable internal translating mass. Translating mass control has a significant advantage over other types of control mechanisms, such as canards or pulse jets, in that all components of the control mechanism are internal to the projectile. In addition, translating mass actuators can be implemented using simple and robust hardware, resulting in a mechanism less susceptible to damage during launch. While internal translating mass control has been considered for many types of projectiles, its use on mortars is particularly attractive due to a mortar's low spin rate and relatively low static stability, factors that reduce actuator power required and increase control authority.

Control authority of the guided mortar is examined using a flight simulation of an example projectile equipped with an internal translating mass (ITM). This simulation comprises a dynamic model of the projectile coupled to a control system for the ITM. The 7-degree-of-freedom dynamic model consists of 14 scalar differential equations that are integrated forward in time from a given set of initial conditions. The ITM control system utilizes a feedback linearization controller that translates the mass within the projectile in a controlled fashion at a frequency equal to the projectile roll rate. While an outline of the dynamic model is presented here, a full derivation of the equations of motion and expressions for the feedback linearization controller can be found in Rogers and Costello.¹

This report outlines several example cases and trade studies to demonstrate the capabilities of translating mass control with the 81-mm mortar. Configuration parameters such as mass size, ITM position within the round, and ITM oscillation amplitude are all varied to evaluate their impact on system performance. Furthermore, two different static stability configurations are included in the analysis in order to show the substantial control authority improvements when projectile static stability is reduced.

2. Dynamic Model

The system consists of two major components, namely, a main projectile body and an internal translating mass. The main projectile body is largely a typical mortar projectile with the exception of an internal cavity that hosts an internal mass. The internal mass is free to translate within the main projectile cavity. An actuator system inside the projectile exerts a force on the

¹Rogers, J.; Costello, M. Control Authority of a Projectile Equipped With an Internal Translating Mass. *Journal of Guidance, Control, and Dynamics* **2008**, *31* (5), 1323–1333.

internal mass as well as the main projectile to move the mass inside the cavity to a desired location. A schematic of the 81-mm mortar projectile equipped with an ITM is shown in figure 1, and figure 2 shows the earth-fixed and body-fixed coordinate systems.

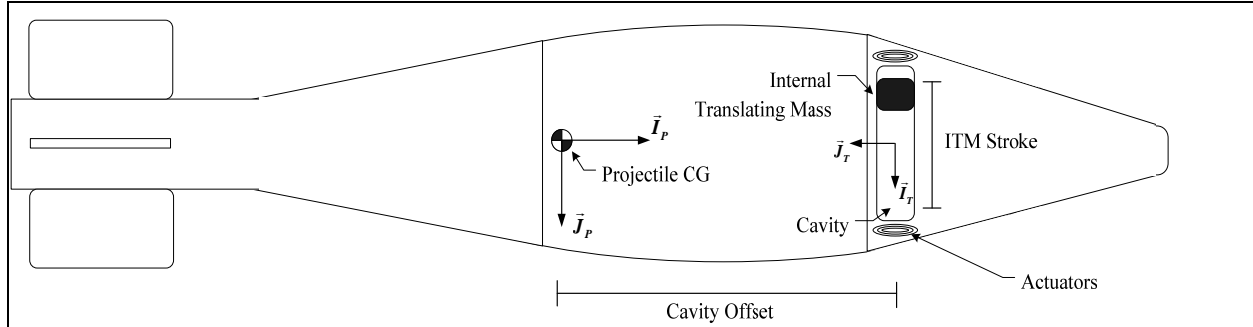


Figure 1. Top down view of 81-mm mortar equipped with an ITM.

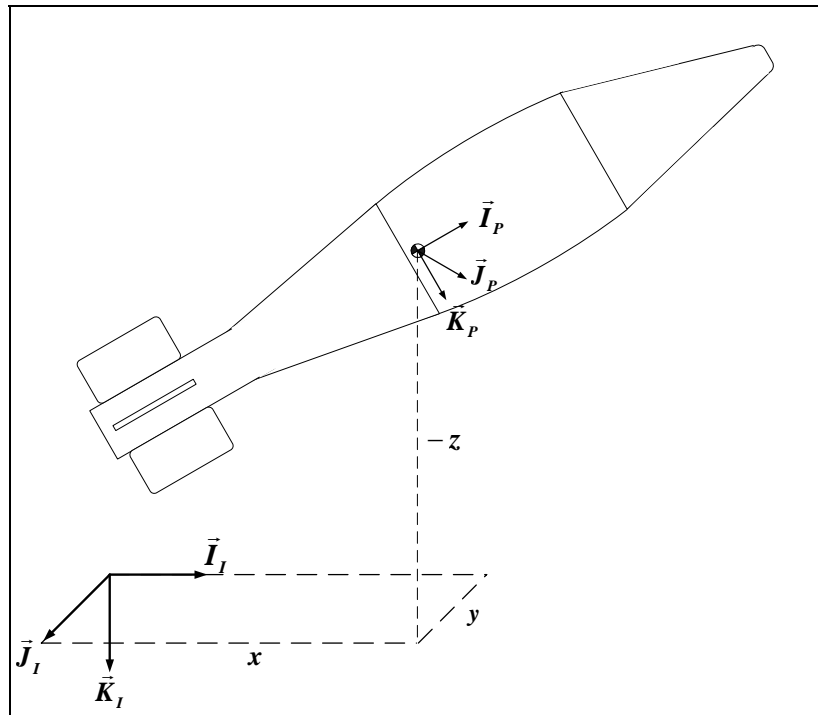


Figure 2. Position coordinates schematic of the 81-mm mortar projectile.

Section 2.1 describes the dynamic equations of the ITM-mortar system, while section 2.2 outlines the physical parameters of the example projectile. Section 2.3 describes the ITM control setup and configuration parameters used throughout the results section.

2.1 Dynamic Equations

The dynamic equations for this 7-degree-of-freedom system consist of a set of kinematic, translational, and rotational dynamic equations. The reader is referred to Rogers and Costello¹ for a complete derivation of the dynamic equations for a projectile equipped with an internal translating mass. Only the final results of the derivation are summarized in this report.

Six kinematic equations, relating position states to velocity states, are identical to those for a 6-degree-of-freedom rigid projectile and are given by equations 1 and 2.

$$\begin{Bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} = \begin{bmatrix} c_\theta c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi \\ c_\theta s_\psi & s_\phi s_\theta s_\psi + c_\phi c_\psi & c_\phi s_\theta s_\psi - s_\phi c_\psi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}. \quad (1)$$

$$\begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix} = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{bmatrix} \begin{Bmatrix} p \\ q \\ r \end{Bmatrix}. \quad (2)$$

An additional kinematic equation is given by the trivial relationship

$$\dot{s} = v_s, \quad (3)$$

which relates the time derivative of the ITM position to its velocity with respect to the projectile. Three translational dynamic equations are also identical to those used to describe rigid projectile motion, given by

$$\begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{Bmatrix} = \begin{Bmatrix} \frac{X}{m} \\ \frac{Y}{m} \\ \frac{Z}{m} \end{Bmatrix} - \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}. \quad (4)$$

Another translational dynamic equation incorporates the acceleration of the translating mass and is given by

$$\begin{bmatrix} A_{S1} & A_{S2} & A_{S3} \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \ddot{s} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{Bmatrix} = \{B_S\} , \quad (5)$$

where

$$A_{S1} = m_T \begin{bmatrix} c_{\theta_T} c_{\psi_T} , & c_{\theta_T} s_{\psi_T} , & -s_{\theta_T} \end{bmatrix} , \quad (6)$$

$$A_{S2} = \frac{m_P m_T}{m} , \quad (7)$$

$$A_{S3} = -\frac{m_P m_T}{m} \begin{bmatrix} c_{\theta_T} c_{\psi_T} , & c_{\theta_T} s_{\psi_T} , & -s_{\theta_T} \end{bmatrix} \mathbb{S}_P(\vec{r}_{P \rightarrow T}) , \quad (8)$$

and

$$\begin{aligned} B_S = & f_{Input} - c_V \dot{s} \\ & + m_T g (-s_{\theta} c_{\theta_T} c_{\psi_T} + s_{\phi} c_{\theta} c_{\theta_T} s_{\psi_T} - c_{\phi} c_{\theta} s_{\theta_T}) \\ & - m_T \begin{bmatrix} c_{\theta_T} c_{\psi_T} , & c_{\theta_T} s_{\psi_T} , & -s_{\theta_T} \end{bmatrix} \mathbb{S}_P(\vec{\omega}_{P/I}) \mathbb{C}_P(\vec{v}_{C/I}) \\ & - \frac{m_P m_T}{m} \begin{bmatrix} c_{\theta_T} c_{\psi_T} , & c_{\theta_T} s_{\psi_T} , & -s_{\theta_T} \end{bmatrix} \mathbb{S}_P(\vec{\omega}_{P/I}) \mathbb{S}_P(\vec{\omega}_{P/I}) \mathbb{C}_P(\vec{r}_{P \rightarrow T}) . \end{aligned} \quad (9)$$

Expressed in the projectile reference frame, the components of the rotational kinetic differential equations are

$$A_{RR} \begin{Bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{Bmatrix} + A_{RS} \{\ddot{s}\} = \{B_R\} , \quad (10)$$

where

$$A_{RR} = I_P + I_T - \frac{m_T}{m_P} m \mathbb{S}_P(\vec{r}_{C \rightarrow T}) \mathbb{S}_P(\vec{r}_{C \rightarrow T}) , \quad (11)$$

$$A_{RS} = m_T \mathbb{S}_P(\vec{r}_{C \rightarrow T}) [T_{PT}]^T , \quad (12)$$

and

$$B_R = \begin{Bmatrix} M_X \\ M_Y \\ M_Z \end{Bmatrix} - \mathbb{S}_P(\vec{\omega}_{P/I}) \left(I_P + I_T - \frac{m_T}{m_P} m \mathbb{S}_P(\vec{r}_{C \rightarrow T}) \mathbb{S}_P(\vec{r}_{C \rightarrow T}) \right) \mathbb{C}_P(\vec{\omega}_{P/I}) - 2m_T \mathbb{S}_P(\vec{r}_{C \rightarrow T}) \mathbb{S}_P(\vec{\omega}_{P/I}) [T_{PT}]^T \begin{Bmatrix} \dot{s} \\ 0 \\ 0 \end{Bmatrix}. \quad (13)$$

The dynamic equations of motion for the internal translating mass projectile are collectively given by equations 1–5 and 10. With a known set of initial conditions, these 14 scalar equations are numerically integrated forward in time using a fourth-order Runge-Kutta algorithm to obtain a single trajectory. The standard aerodynamic expansion employed for projectile flight dynamic simulation is used here, with aerodynamic coefficients generated from PRODAS. A full description of the weight force and body aerodynamic forces and moments is provided in Rogers and Costello.¹

2.2 Example Projectile

The example projectile is based on the M821 81-mm mortar. Table 1 summarizes dimensional and inertial properties of the projectile.

Table 1. Properties of the example mortar projectile.

Diameter	80.7 mm
Projectile Mass	4.583 kg
Axial Inertia	0.00371 kg-m ²
Transverse Inertia	0.04993 kg-m ²
CG Stationline (Nominal)	0.282 m
Fin Cant Angle	1.4°

Note: CG = center of gravity.

Note that the CG stationline position is listed as “nominal,” since other CG positions are considered in the results section as well. Initial conditions, used for all simulations throughout the results section, are given in table 2.

Table 2. Initial conditions for the example mortar projectile.

x	0.0 m	u	294.0 m/s
y	0.0 m	v	0.0 m/s
z	0.0 m	w	0.0 m/s
ϕ	0.0 rad	p	0.0 rad/s
θ	0.80 rad	q	0.0 rad/s
ψ	0.0 rad	r	0.0 rad/s

2.3 Internal Translating Mass Control Setup

For all cases considered here, the cavity that contains the ITM is aligned perpendicular to the projectile axis of symmetry, and therefore $\theta_T = 0$ and $\psi_T = \pi/2$ rad. The ITM is oscillated at the projectile roll frequency according to

$$s = A \cos(\phi), \quad (14)$$

where s is the ITM distance from the center of the cavity, A is the ITM oscillation amplitude (exactly half of the ITM stroke), and ϕ is the projectile roll angle. Figure 3 shows a diagram of how translating mass motion occurs through different portions of the projectile roll cycle.

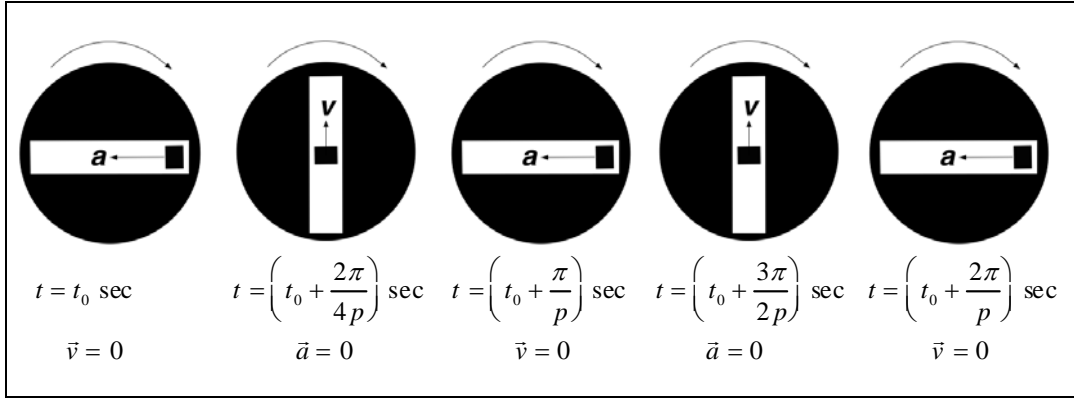


Figure 3. Moving mass position, velocity (\vec{v}) and acceleration (\vec{a}) over one roll cycle, viewed from the rear of the projectile. Mass motion is sinusoidal and has the same frequency as the projectile roll rate.

ITM oscillations are initiated as soon as the projectile is launched and result in full control authority in the inertial frame negative y direction (to the left when looking downrange). Two sets of ITM configuration parameters, given in table 3, are studied in order to determine how different variables affect control authority.

Table 3. ITM configuration parameters for example trajectories.

Projectile Physical Variable	Parameter Set A	Parameter Set B
ITM mass size (g)	125	125
ITM stroke (cm)	1.27	5.08
Projectile CG stationline (m from base)	0.282	0.209
Cavity offset from CG (m)	0.167	0.167

Parameter set A represents the configuration originally proposed by the U.S. Army Research Laboratory, while parameter set B represents a projectile with reduced stability (CG farther aft) and a significant increase in ITM oscillation amplitude (“stroke”). A stroke of 5.08 cm was determined to be approximately the maximum amplitude possible given the width of the round at that cavity offset. Also, note that a CG stationline of 0.209 m, used in parameter set B, ensures a static margin of at least 0.25 cal. throughout the flight.

3. Results

Both example cases and trade studies were performed to demonstrate the capabilities of ITM control. Section 3.1 outlines example controlled simulations using both sets of configuration parameters, as well as uncontrolled cases in order to establish a baseline for comparison. Section 3.2 includes trade studies that examine the effect of ITM size, stroke, and cavity offset on control authority.

3.1 Example Trajectories

Several controlled trajectories are simulated and compared to uncontrolled simulations (in which the mass is present but not oscillated). As described in section 2.3, the controller commands mass oscillation to begin immediately after launch and continues oscillation throughout the remainder of flight. The oscillation function given in equation 14 corresponds to control applied in the inertial frame negative y direction (see figure 2). Furthermore, the set of initial conditions given in table 2 corresponds to approximately maximum range for the example mortar projectile. Therefore, all controlled trajectories show maximum control authority in deflection for a specific ITM configuration at maximum range. This maximum deflection capability is used as the quantitative measure of control authority.

Figures 4–10 show four example trajectories. Two uncontrolled trajectories are shown, one for a projectile of nominal stability (CG position of 0.282 m) and one for a projectile of reduced stability (CG position of 0.209 m). Note that these values correspond to those used in ITM parameter sets A and B, respectively. Several states of the reduced stability uncontrolled trajectory have been excluded since they are nearly identical to those of the nominal stability uncontrolled case. Two controlled trajectories are also shown, one using parameter set A and the other using parameter set B.

The most significant plots are figures 5, deflection vs. range, and 10, angle of attack vs. time. Both show that the ITM control mechanism has little effect for the configuration given by parameter set A. Maximum control authority for this case yields deflection of ~1 m from nominal at maximum range and a trim angle of attack of $<0.1^\circ$. However, parameter set B shows significantly better results, with a deflection of over 20 m from nominal at maximum range and a trim angle of attack of $\sim 0.3^\circ$. Figure 5 also shows trajectory deflections for the uncontrolled rounds. This is due to drift, which is caused by an interaction between the projectile rolling and pitching dynamics with the gravity-induced trajectory curvature. Drift is larger for the reduced stability configuration than for the nominal stability case due to a decrease in restoring aerodynamic moment.

Figures 5 and 10 highlight the fact that ITM systems typically only show good performance in rounds with reduced static stability, where ITM size and stroke are as large as possible.

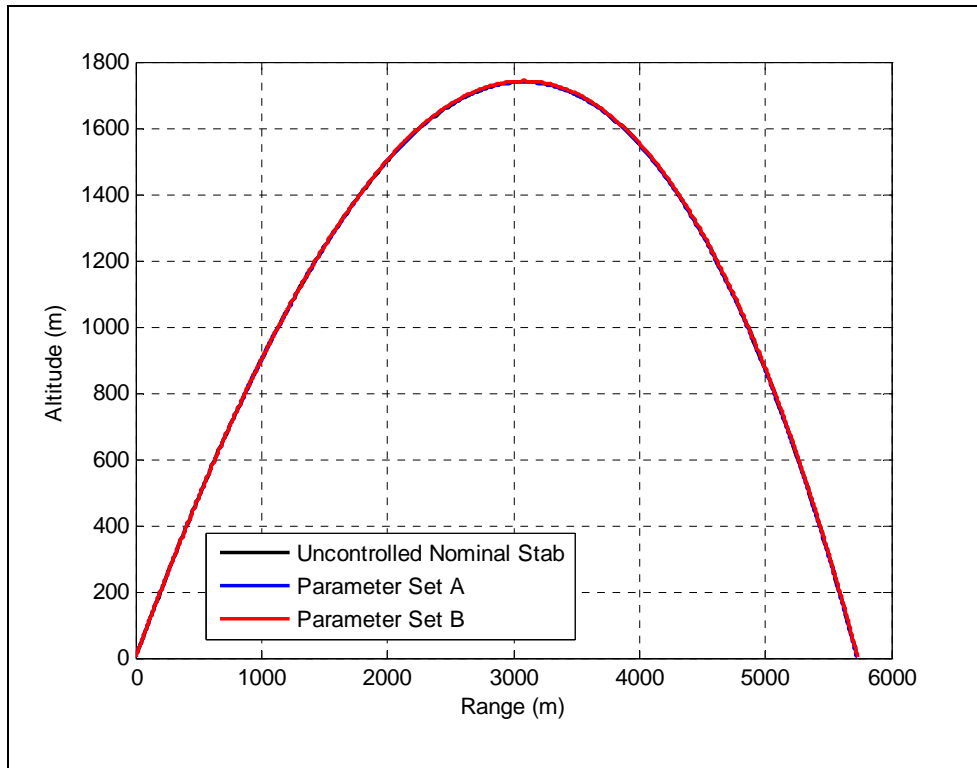


Figure 4. Altitude vs. range. All trajectories overlay one another.

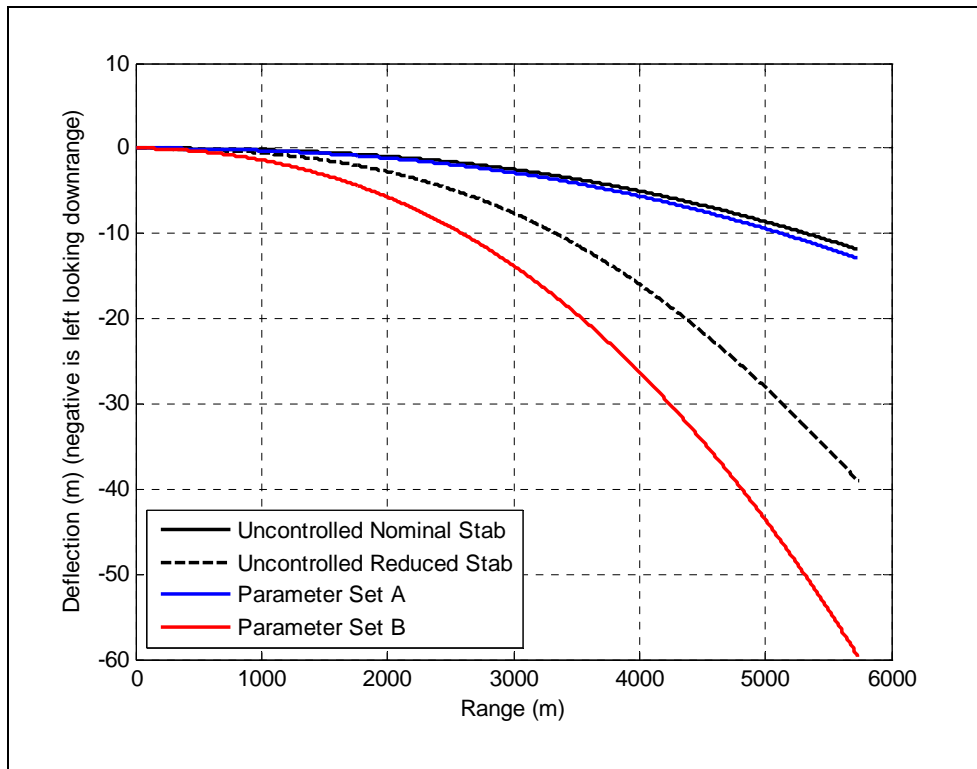


Figure 5. Deflection vs. range.

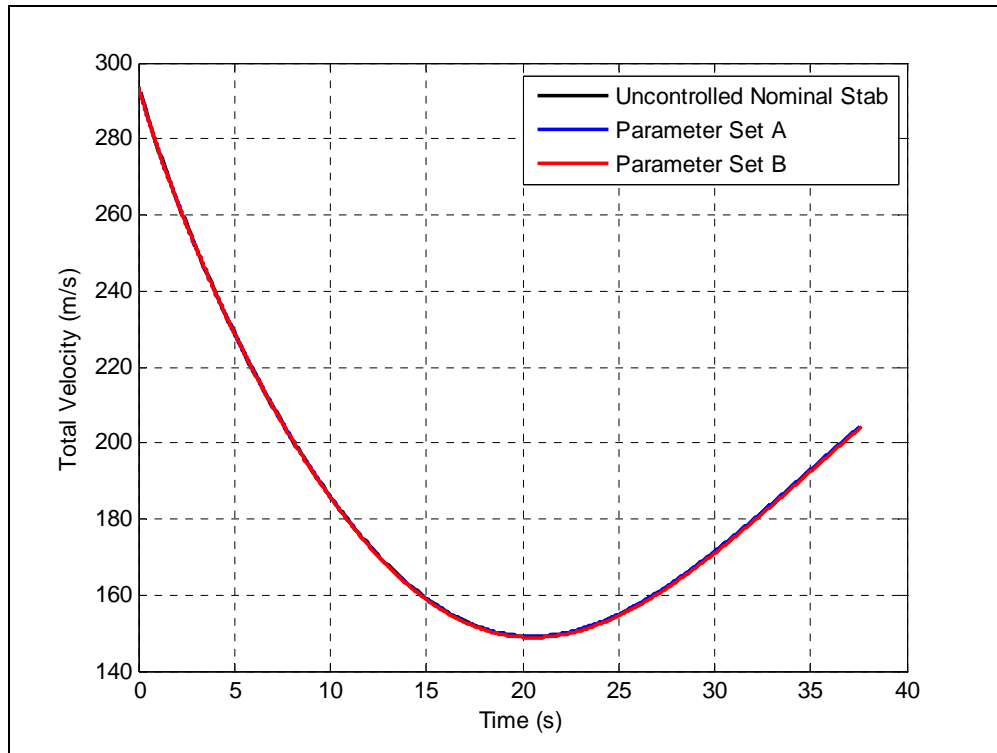


Figure 6. Total velocity vs. time. All trajectories overlay one another.

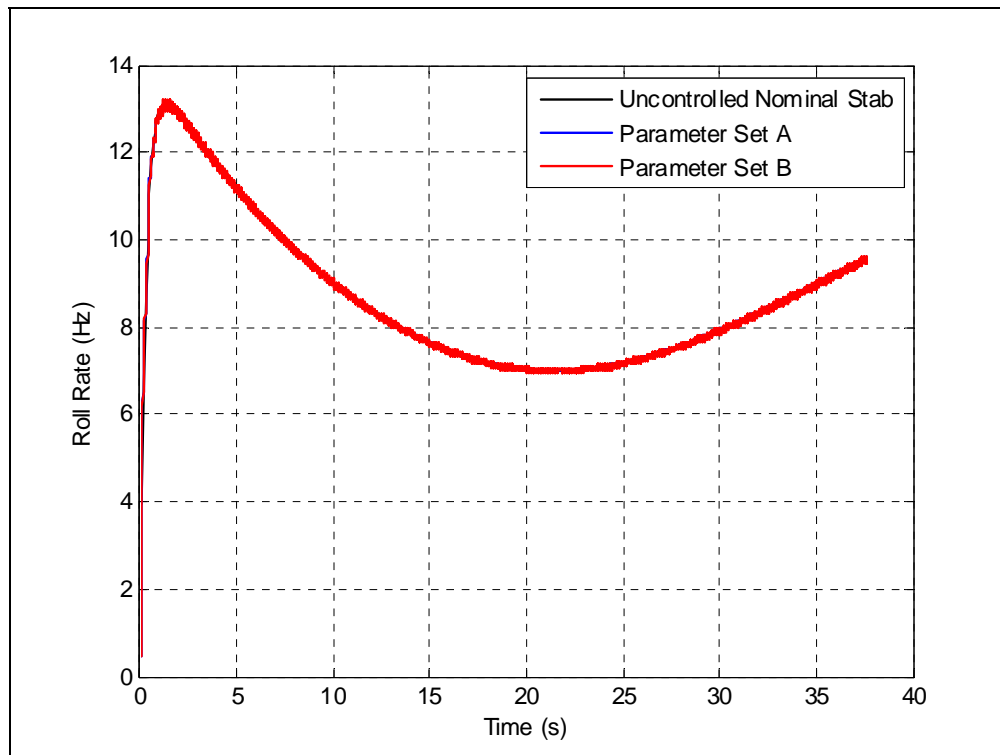


Figure 7. Roll rate vs. time. All trajectories approximately overlay one another.

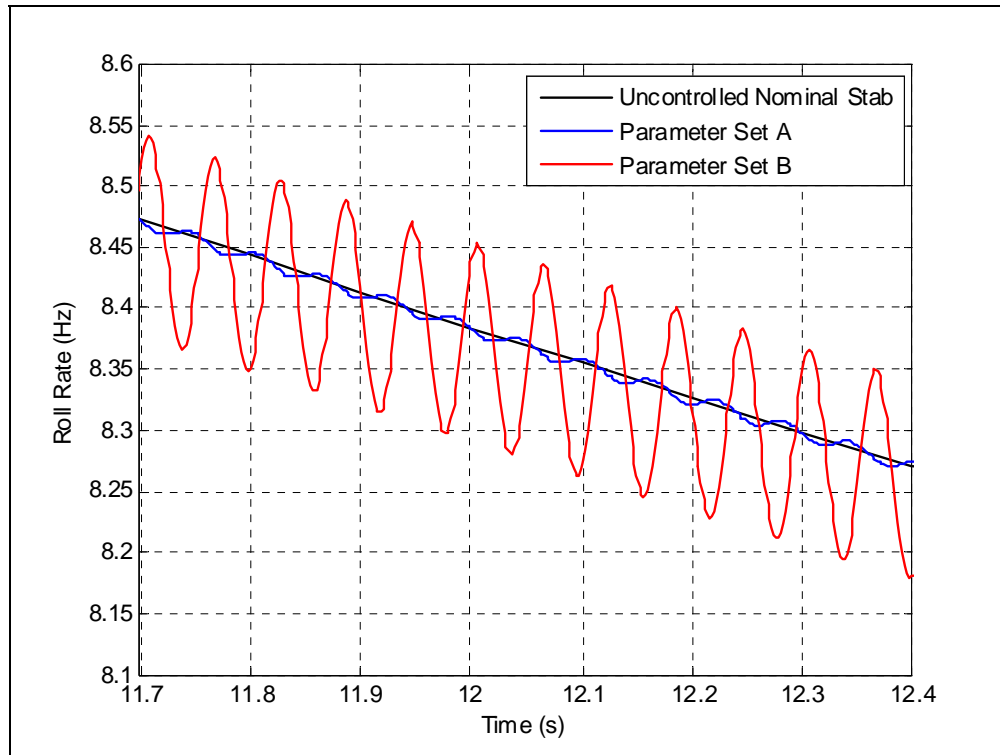


Figure 8. Zoom view of roll rate vs. time. Notice roll rate oscillations as ITM oscillates, changing the axial moment of inertia of the system.

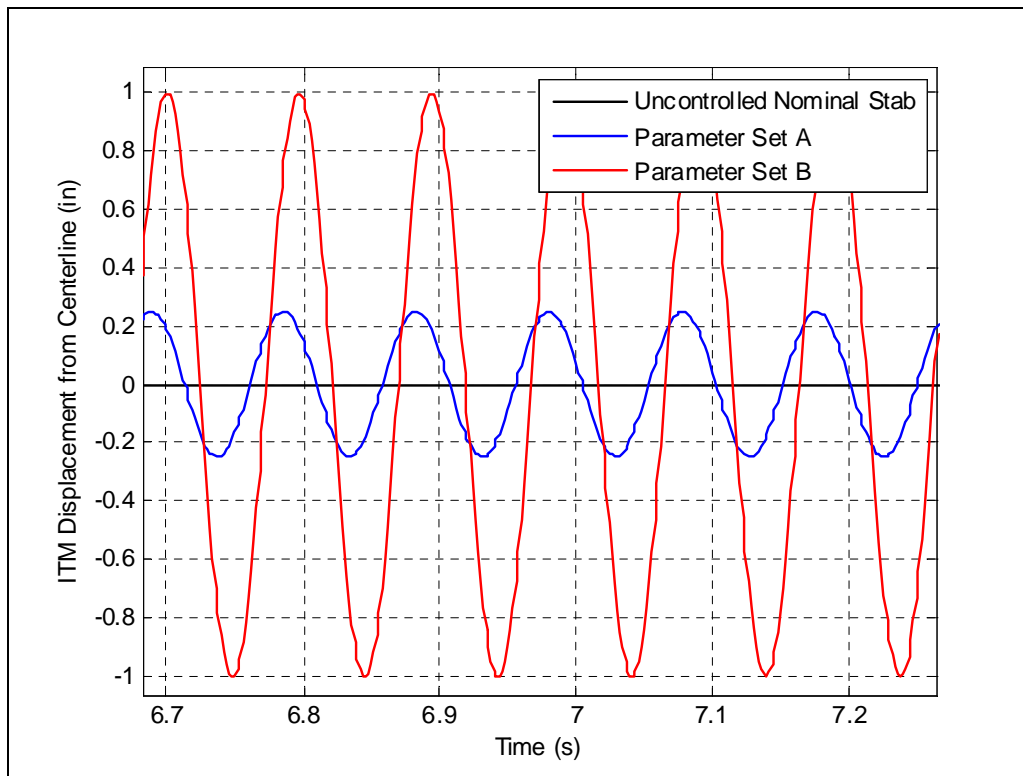


Figure 9. Segment of ITM displacement from centerline vs. time.

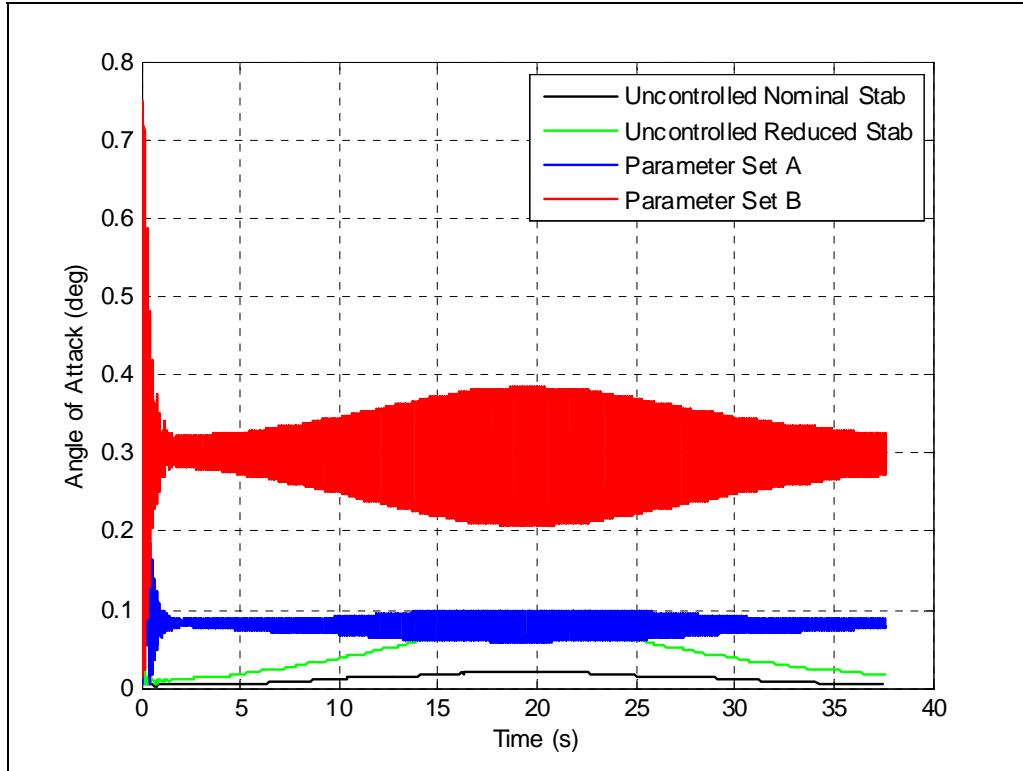


Figure 10. Angle of attack vs. time.

3.2 Trade Studies

A trade study was conducted to examine the effect of cavity offset on ITM control authority. In addition, the ITM stroke was varied for each cavity offset. Simulations utilizing identical initial conditions to the examples in section 3.1 were performed using both nominal stability (CG stationline of 0.282 m) and reduced stability (CG stationline of 0.209 m). Radial deviation from the nominal case was determined for each simulation. Note that radial deviation is calculated as the square root of the sum of the squares of the deviations in deflection and range, which occurs due to a slight off-axis response of the projectile. Figure 11 shows control authority results for the nominal stability cases, while figure 12 shows control authority results for the reduced stability cases. ITM size was 125 g for all simulations. Note that deviation is nearly independent of cavity offset. This is due to the fact that the control mechanism relies on a lateral mass center offset, which can be produced at any stationline on the projectile. Slight increases in radial deviation seen with larger offsets are the result of drag penalties associated with higher angle of attack oscillations at large cavity offsets. Also note that not all combinations of ITM stroke and cavity offset are physically feasible given the constraints of the 81-mm round. However, the results show important trends, specifically that only large ITM stroke combined with reduced stability can produce any useful control authority.

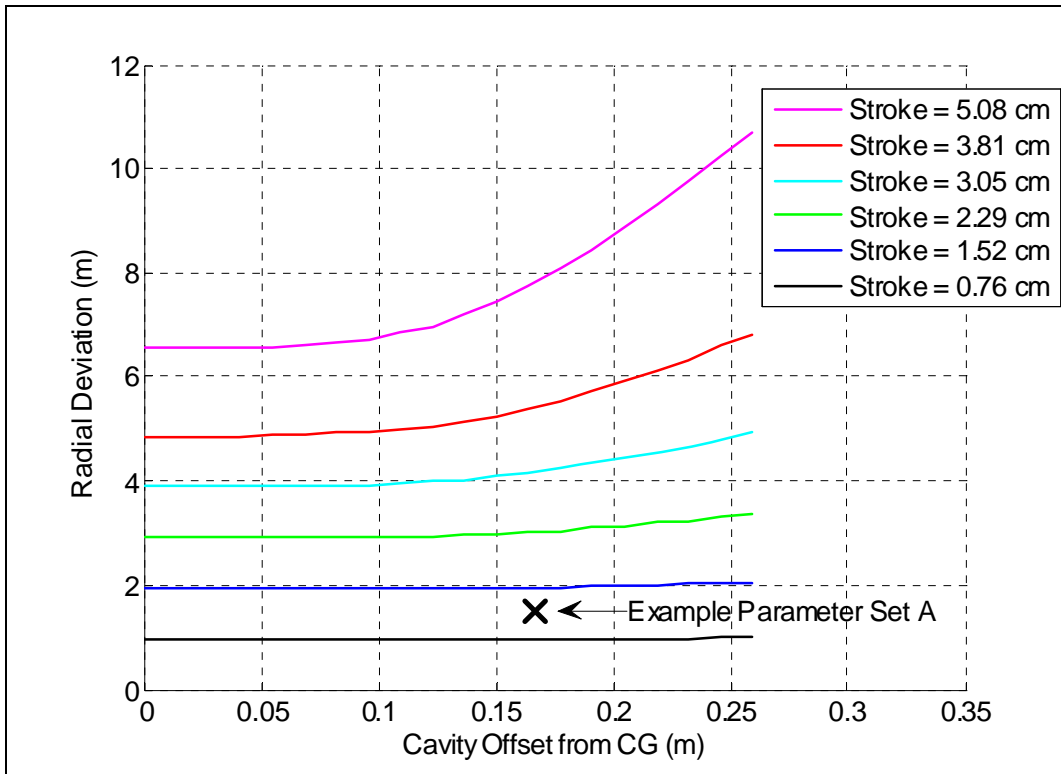


Figure 11. Radial deviation from standard trajectory vs. cavity offset for nominal stability (ITM mass of 125 g, CG stationline of 0.282 m).

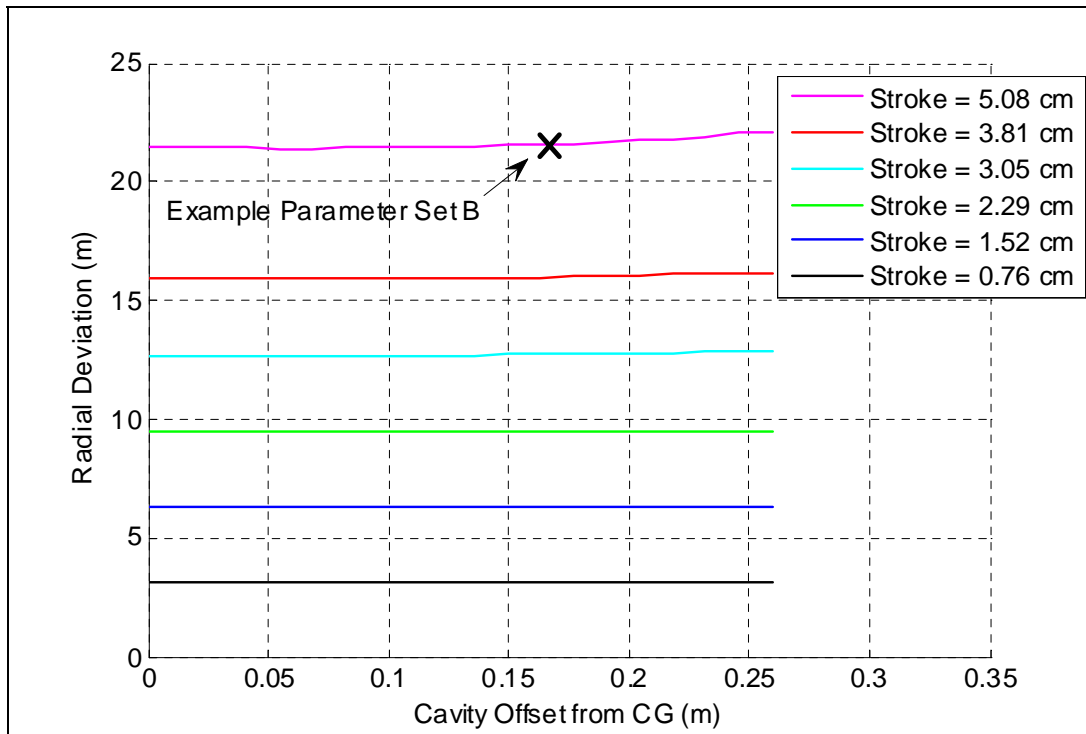


Figure 12. Radial deviation from standard trajectory vs. cavity offset for reduced stability (ITM mass of 125 g, CG stationline of 0.209 m).

Similar trade studies were conducted by fixing the cavity offset at 0.167 m and varying the ITM stroke and mass size. Again, the study was conducted for both the nominal and reduced stability cases. Figure 13 shows control authority results for the nominal stability cases, while figure 14 shows control authority results for the reduced stability cases.

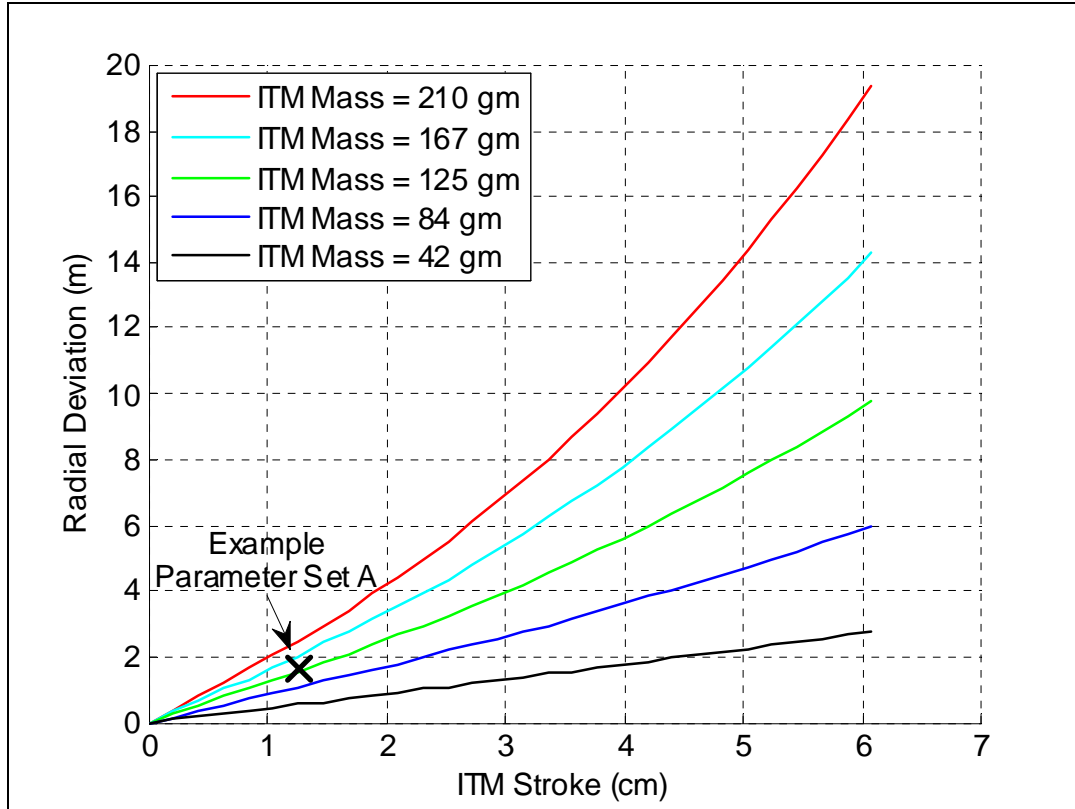


Figure 13. Radial deviation from standard trajectory vs. ITM stroke for nominal stability (cavity offset of 0.167 m, CG stationline of 0.282 m).

Figures 13 and 14 again demonstrate that the reduced stability ITM projectile exhibits significantly better performance. A larger stroke and larger mass produce much greater control authority. However, it is important to note that larger mass sizes and oscillation amplitudes require larger actuators and more battery power.

4. Conclusion

A control authority study of an 81-mm mortar equipped with a controllable internal translating mass was presented. Example results show that reasonable control authority is achieved only in the case of decreased static stability. Parametric trade studies further showed that control authority increases as mass size and oscillation amplitude increase, and is insensitive to cavity offset.

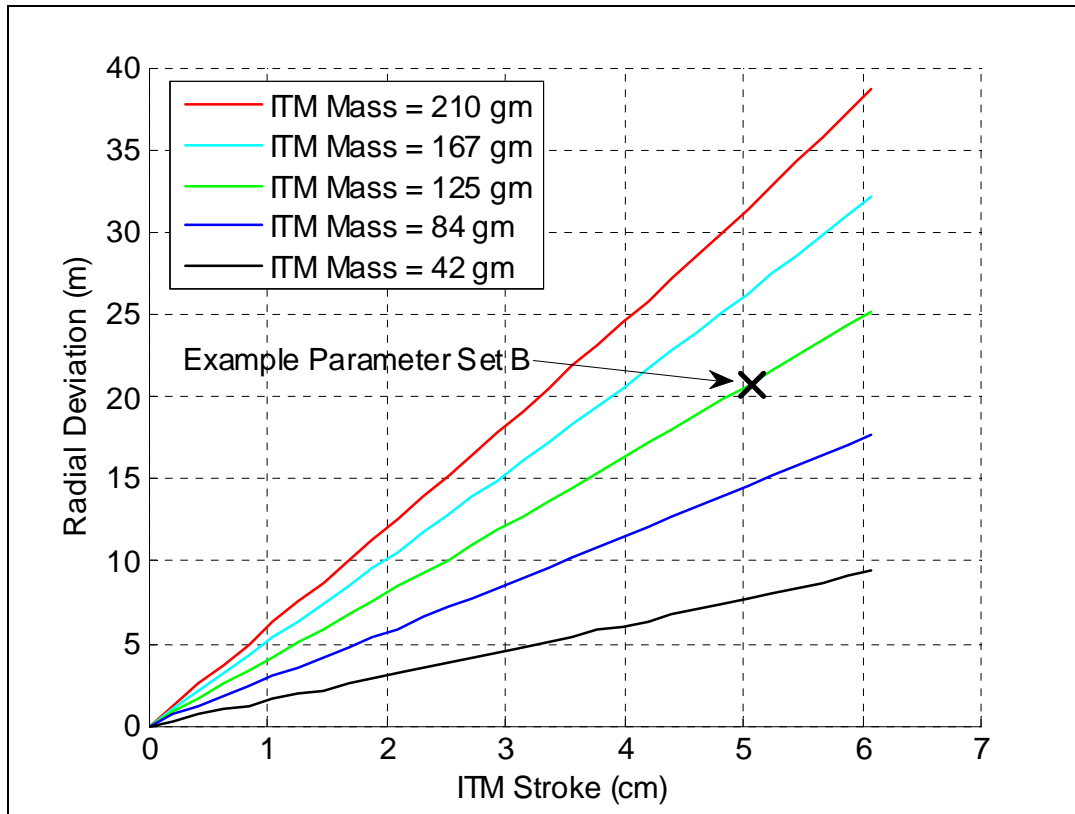


Figure 14. Radial deviation from standard trajectory vs. ITM stroke for reduced stability (cavity offset of 0.167 m, CG stationline of 0.209 m).

Analysis of these results leads to the conclusion that use of an ITM mechanism results in far less control authority than is possible with a mechanism such as canards, which can achieve hundreds of meters of swerve in deflection at maximum range. Furthermore, in order to create reasonable control authority with the ITM mechanism, the projectile static margin would need to be substantially reduced to ~ 0.25 cal. or less. Therefore, since the goal of controlled munitions is often primarily to reduce or eliminate trajectory dispersion, it is highly unlikely that an 81-mm mortar equipped with an ITM control system would be capable of control maneuvers needed to eliminate launch errors and trajectory disturbances.

List of Symbols, Abbreviations, and Acronyms

c_γ	The cosine of argument angle γ .
CG	Center of gravity.
f_{Input}	Scalar value of the input force exerted by the controller.
g	The acceleration due to gravity (32.2 ft/s ²).
I_P	Mass moment of inertia matrix of the projectile body with respect to the projectile reference frame.
I_T	Mass moment of inertia matrix of the internal translating mass with respect to the projectile reference frame.
ITM	Internal translating mass.
$\vec{I}_I, \vec{J}_I, \vec{K}_I$	Inertial reference frame unit vectors.
$\vec{I}_P, \vec{J}_P, \vec{K}_P$	Projectile reference frame unit vectors.
$\vec{I}_T, \vec{J}_T, \vec{K}_T$	Internal translating mass reference frame unit vectors.
m_P	Projectile body mass.
m_T	Internal translating mass.
m	Total system mass.
M_X, M_Y, M_Z	External moment components on the projectile body expressed in the projectile reference frame.
p, q, r	Components of the angular velocity vector of the projectile body expressed in the projectile reference frame.
$\vec{r}_{C \rightarrow T}$	Distance vector from the center of mass of the system to the internal translating mass center of mass.

$\vec{r}_{P \rightarrow T}$	Distance vector from the projectile center of mass to the internal translating mass center of mass.
s	Position of the internal translating mass along its line of movement with respect to the center of the cavity.
s_γ	The sine of argument angle γ .
$[T_{PT}]$	Transformation matrix from the projectile reference frame to the translating mass frame.
t_γ	The tangent of argument angle γ .
u, v, w	Translation velocity components of the composite body center of mass resolved in the projectile reference frame.
$\vec{v}_{C/I}$	Velocity of the system mass center with respect to the inertial frame.
v_s	Magnitude of the velocity of the translating mass with respect to the translating mass reference frame.
x, y, z	Position vector components of the composite body center of mass expressed in the inertial reference frame.
X, Y, Z	Total external force components on the composite body expressed in the projectile reference frame.
ϕ, θ, ψ	Euler roll, pitch, and yaw angles.
θ_T, ψ_T	Euler pitch and yaw angles for the orientation of the line of movement of the internal translating mass with respect to the projectile body.
$\vec{\omega}_{P/I}$	Angular velocity of the projectile body with respect to the inertial frame.
$\mathbb{C}_X(\vec{y})$	Vector component operator that outputs a column vector comprising the components of the input vector \vec{y} expressed in reference frame \mathbf{X} .
$\mathbb{S}_X(\vec{y})$	Cross product operator that outputs a skew-symmetric matrix using the components of the input vector \vec{y} expressed in reference frame \mathbf{X} .

NO. OF
COPIES ORGANIZATION

1 DEFENSE TECHNICAL
 (PDF INFORMATION CTR
 only) DTIC OCA
 8725 JOHN J KINGMAN RD
 STE 0944
 FORT BELVOIR VA 22060-6218

1 DIRECTOR
 US ARMY RESEARCH LAB
 IMNE ALC HRR
 2800 POWDER MILL RD
 ADELPHI MD 20783-1197

1 DIRECTOR
 US ARMY RESEARCH LAB
 AMSRD ARL CI OK TL
 2800 POWDER MILL RD
 ADELPHI MD 20783-1197

1 DIRECTOR
 US ARMY RESEARCH LAB
 AMSRD ARL CI OK PE
 2800 POWDER MILL RD
 ADELPHI MD 20783-1197

ABERDEEN PROVING GROUND

1 DIR USARL
 AMSRD ARL CI OK TP (BLDG 4600)

NO. OF
COPIES ORGANIZATION

4 GEORGIA TECH RSCH INST
J MCMICHAEL
K MASSEY
A LOVAS
M HEIGES
7220 RICHARDSON RD
SMYRNA GA 30080

1 USAMCOM
AMSAM RD MG
J BAUMANN
REDSTONE ARSENAL AL 35898-5000

1 USAMCOM
AMSRD AMR SG SD
B NOURSE
REDSTONE ARSENAL AL 35898-5000

1 AFRL/MNGN
G ABATE
101 W EGLIN BLVD STE 332
EGLIN AFB FL 32542

2 GEORGIA TECH
DEPT OF ARSPC
ENGINEERING
M COSTELLO
J ROGERS
270 FERST DR
ATLANTA GA 30332-0150

1 US AMRDEC
AMSAM RD SS AT
R KRETZSCHMAR
BLDG 5400
REDSTONE ARSENAL AL 35898-5000

1 US RDECOM
AMRDEC
AMSRD AMR SG CT
S DUNBAR
BLDG 5400
REDSTONE ARSENAL AL 35898

1 US RDECOM
AMSRD AMR SG SD
J LOCKER
BLDG 5400
REDSTONE ARSENAL AL 35898-5000

NO. OF
COPIES ORGANIZATION

4 COMMANDER ARDEC
AMSRD AAR AEM L
D OKKEN
R BRYAN
M HORVATH
P BRISLIN
BLDG 65 S
PICATINNY ARSENAL NJ 07806-5000

2 COMMANDER ARDEC
AMSRD AAR AEM
M LUCIANO
M PALATHINGAL
BLDG 65 S
PICATINNY ARSENAL NJ 07806-5000

1 COMMANDER ARDEC
AMSRD AAR AEP E
C STOUT
BLDG 94
PICATINNY ARSENAL NJ 07806-5000

1 COMMANDER ARDEC
AMSRD AAR AEM J
G FLEMING
BLDG 65N
PICATINNY ARSENAL NJ 07806-5000

1 COMMANDER ARDEC
AMSRD AAR AEM D
G MOSHIER
BLDG 65N
PICATINNY ARSENAL NJ 07806-5000

1 COMMANDER ARDEC
AMSRD AAR AIS SA
D ERICSON
BLDG 12
PICATINNY ARSENAL NJ 07806-5000

1 PROJECT MGR
TANK MAIN ARMAMENT SYS
SFAE GCSS TMA
D GUZIEWICZ
BLDG 354
PICATINNY ARSENAL NJ 07806-5000

1 APM SMALL & MEDIUM CALIBER
AMMO OPM-MAS
SFAE GCSS TMA
G DEROSA
BLDG 354
PICATINNY ARSENAL NJ 07806-5000

NO. OF
COPIES ORGANIZATION

1 COMMANDER
US ARMY TACOM ARDEC
SFAE GCSS MAS SMC
R KOWALSKI
BLDG 354
PICATINNY ARSENAL NJ 07806-5000

2 COMMANDER
US ARMY TACOM ARDEC
AMSTA AR FSP P
P MAGNOTTI
A LICHTENBERG-SCANLON
BLDG 61S
PICATINNY ARSENAL NJ 07806-5000

1 COMMANDER
US ARMY TACOM ARDEC
AMSRD AAR
A MUSALLI
BLDG 61S
PICATINNY ARSENAL NJ 07806-5000

2 COMMANDER
US ARMY TACOM ARDEC
AMSRD AAR AEM L
G KOLASA
A MOUNA
BLDG 61S
PICATINNY ARSENAL NJ 07806-5000

11 COMMANDER
US ARMY TACOM ARDEC
AMSRD AAR AEM A
G LIVECCHIA
J GRAU
G MALEJKO
E VAZQUEZ
W TOLEDO
L YEE
R TROHANOWSKY
S HAN
W KOENIG
S CHUNG
C WILSON
BLDG 95
PICATINNY ARSENAL NJ 07806-5000

1 COMMANDER
US ARMY TACOM ADRDEC
AMSRD AAR AEM L
R SAYER
BLDG 65
PICATINNY ARSENAL NJ 07806-5000

NO. OF
COPIES ORGANIZATION

1 COMMANDER
US ARMY TACOM ADRDEC
SFAE AMO MAS LC
F CHANG
BLDG 354
PICATINNY ARSENAL NJ 07806-5000

1 COMMANDER
US ARMY TACOM ADRDEC
AMSTA AR CCH B
S PATEL
BLDG 65
PICATINNY ARSENAL NJ 07806-5000

1 COMMANDER
US ARMY TACOM
SFAE GCSS W AB QT
J MORAN
WARREN MI 48397-5000

3 COMMANDER
US ARMY TACOM
SFAE GCSS W
S COOPER
J NEFF
J FLECK
WARREN MI 48397-5000

1 COMMANDER
US ARMY TACOM
SFAE CSS TV
R GROLLER
WARREN MI 48397-5000

1 DIRECTOR
US ARMY ARMOR CTR
ATZK TS
W MEINSHAUSEN
FT KNOX KY 40121

2 SFSJM-CDL
HQ US ARMY JOINT MUNITIONS CMD
AMSIO SMT
W HARRIS
M RIVERS
1 ROCK ISLAND ARSENAL
ROCK ISLAND IL 61299-6000

2 DIRECTOR
BENET LABORATORIES
AMSTA AR CCB
J VASILAKIS
R HASENBEIN
WATERVLIET NY 12189

NO. OF
COPIES ORGANIZATION

- 1 APPLIED PHYSICS LAB
W DAMICO
11100 JOHNS HOPKINS RD
LAUREL MD 20723-6099
- 2 ARROW TECH ASSOC INC
M STEINHOFF
W HATHAWAY
1233 SHELBURNE RD
SOUTH BURLINGTON VT 05495
- 1 DIRECTOR
BENET LABORATORIES
AMSTA AR CCB RA
G PFLEGL
WATERVLIET NY 12189
- 1 DIRECTOR
MARINE CORPS PROGRAMS DEPT
NAV SURF WARCENDIC CRANE
M O'MALLEY
700 AMMUNITION RD
FALLBROOK CA 92028-3187
- 1 FIRES DEPUTY MGR
EXPEDITIONARY MANEUVER
WARFARE AND COMBATING
TERRORISM SCI AND
TECHLGY DEPT
OFC OF NVL RSRCH
ONE LIBERTY CTR
875 N RANDOLPH ST
ARLINGTON VA 22203
- 2 NVL SURFC WARFARE CTR
L STEELMAN
PRECISION AND ADVNCD SYS BR
CODE G33
6210 TISDALE RD STE 223
DAHLGREN VA 22448-5114
- 1 NVL SURFC WARFARE CTR
GUN SYS AND LIGHT WPNS DIV
CODE G30
6096 TISDALE RD STE 302
DAHLGREN VA 22448-5156

NO. OF
COPIES ORGANIZATION

ABERDEEN PROVING GROUND

- 19 DIR USARL
AMSRD ARL WM
P PLOSTINS
AMSRD ARL WM B
M ZOLTOSKI
J NEWILL
AMSRD ARL WM BA
D LYON
T BROWN
D HEPNER
J CONDON
AMSRD ARL WM BC
J SAHU
P WEINACHT
B GUIDOS
S SILTON
G OBERLIN
G COOPER
J DESPIRITO
J GARNER
F FRESCONI
B HOWELL
I CELMINS
AMSRD ARL WM S
W CIEPIELA